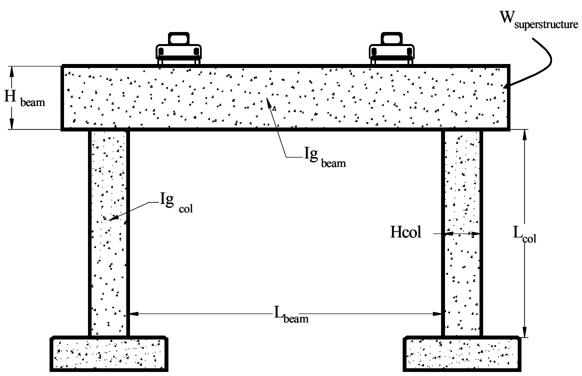
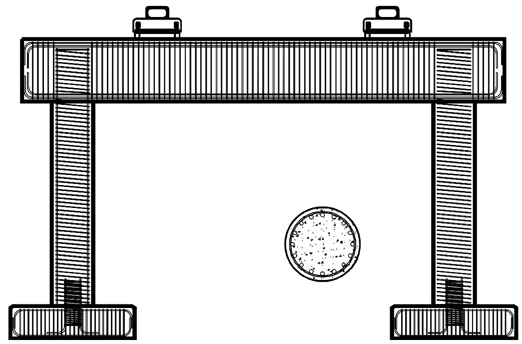
DESIGN/ANALYSIS EXAMPLE
Frame 4 Design

$lb := psi \cdot in^2$ $\quad ksi := 1000 \cdot psi$ $\quad kip := 1000 \cdot lb$ $\quad ii14 := 1 .. 4$ $\quad ii15 := 1 .. 5$

This example assumes that the piers and superstructure of a two-column pin-supported reinforced-concrete bridge bent have been designed and detailed, such that all geometry and reinforcement details are known.





Design structural members framing into joint (columns and beams)

Column length:	$L_{col} := 36 \cdot ft$	Beam length:	$L_{beam} := L_{col}$
Column diameter:	$H_{col} := 6.5 \cdot ft$	Beam depth:	$H_{beam} := 8 \cdot ft$
Column long. steel ratio:	$\rho_{col} := 1.75 \cdot \%$	Beam width:	$B_{beam} := H_{col}$
Column long. steel diameter:	$d_b := 1.693 \cdot in$		
Superstructure Weight:	$Weight := 3000 \cdot kip$		

Concrete Material Properties:

Nominal Compressive Strength: $f_c := 5500 \cdot psi$

Young's modulus of concrete: $E_c := 57000 \cdot \sqrt{f_c \cdot psi}$ $\quad E_c = 4227 \cdot ksi$

Poisson's ratio of concrete: $\nu_c := 0.2$

Shear stiffness modulus: $G_c := \dfrac{E_c}{2 \cdot (1 + \nu_c)}$ $\quad G_c = 1761 \cdot ksi$

Steel Material Properties:

Yield stress of reinforcement: $f_y := 68 \cdot ksi$

Young's modulus of steel $E_s := 29000 \cdot ksi$ $\quad E_s = 29000 \cdot ksi$

Ultimate Steel Strain $\varepsilon_u := 0.1$

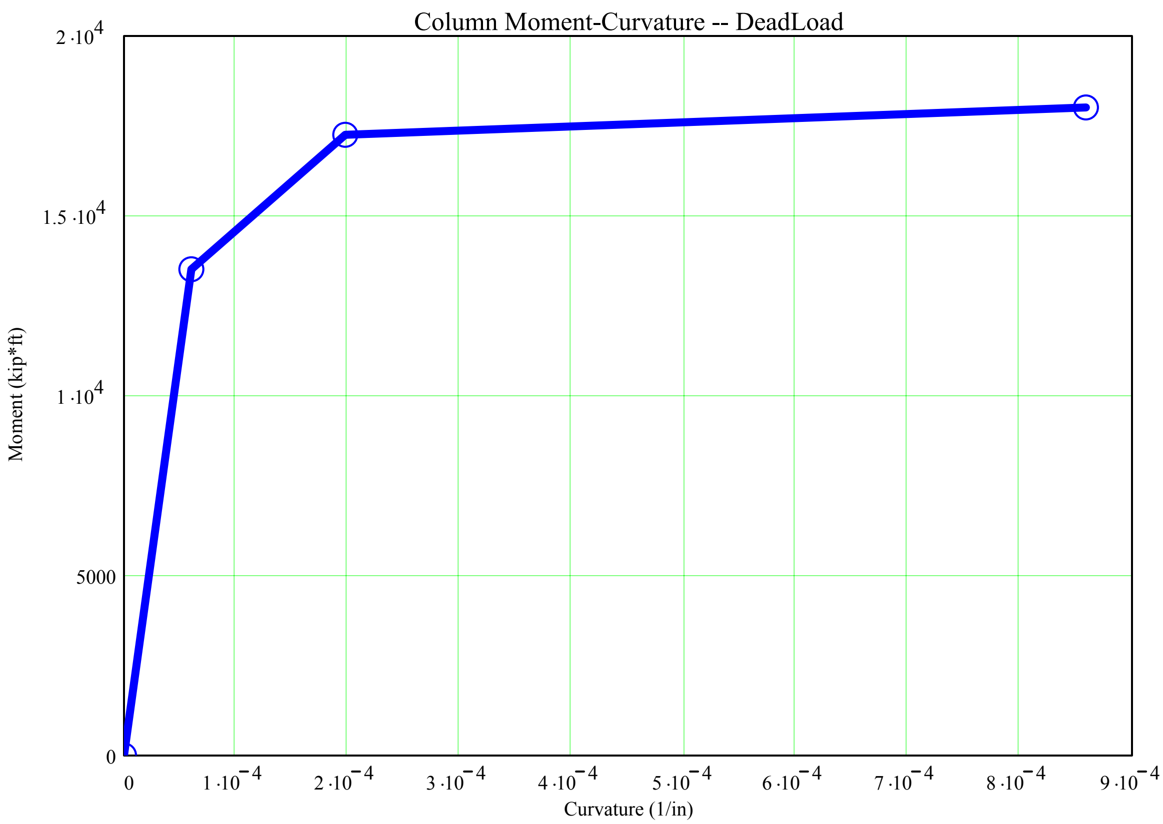
Moment-Curvature Characteristics of Column under dead-load axial load (from section analysis using OpenSees):

First-yield: $\phi y_{col} := 6.0124 \cdot 10^{-5} \cdot \frac{1}{in}$ $My_{col} := 13511 \cdot kip \cdot ft$

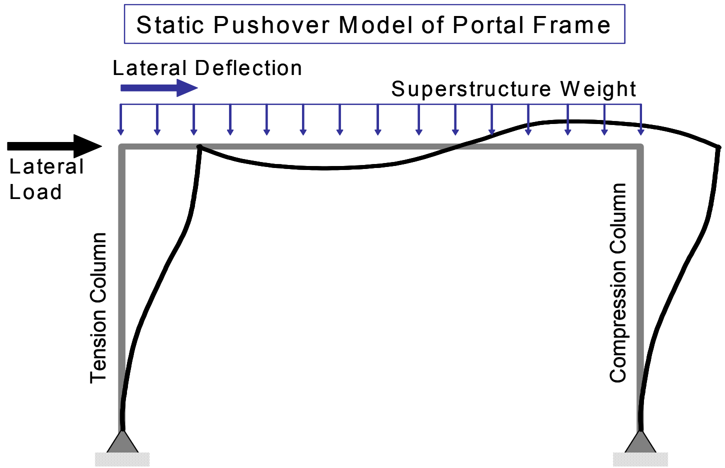
Nominal strength: (extreme compressive strain ε_c=0.003) $\phi n_{col} := 0.00019755 \cdot \frac{1}{in}$ $Mn_{col} := 17248 \cdot kip \cdot ft$

Ultimate strength: (extreme compressive strain ε_c=0.014) $\phi u_{col} := 0.00085891 \cdot \frac{1}{in}$ $Mu_{col} := 18010 \cdot kip \cdot ft$

$\Phi ynu_{col} := \left[0 \cdot \frac{1}{in} \quad \phi y_{col} \quad \phi n_{col} \quad \phi u_{col} \right]^T$ $Mynu_{col} := \left[0 \cdot kip \cdot in \quad My_{col} \quad Mn_{col} \quad Mu_{col} \right]^T$

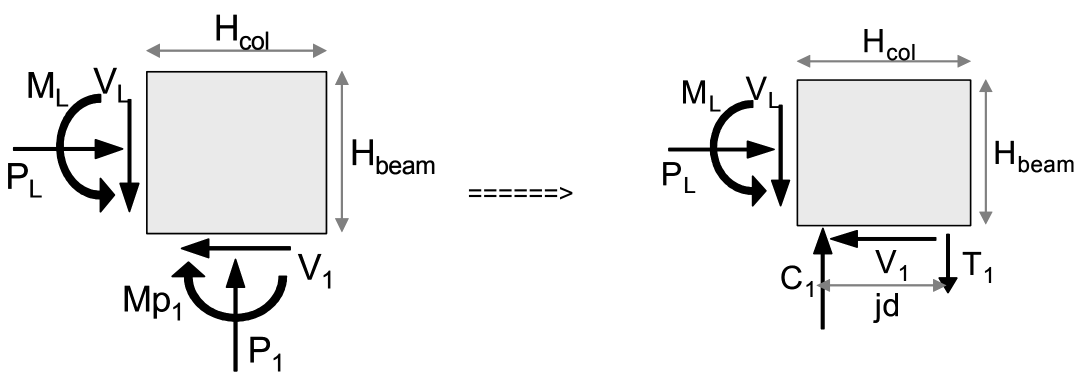


Calculate joint-boundary forces based on equilibrium at maximum moment strength of framing column



Analysis I
estimate joint-boundary forces from Moment-Curvature Data

In the portal frame, the compression column on the right will reach its nominal strength first:

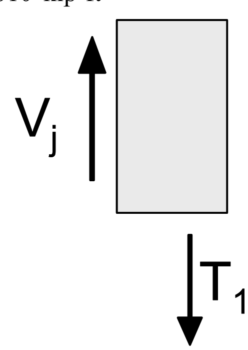


To calculate joint shear, all you really need is the tension component of the moment couple at the joint interface (hence only the column plastic moment):

From M-Φ analysis: $Mp_1 := Mu_{col}$ $\qquad Mp_1 = 18010 \cdot kip \cdot ft$

Assume a value for the moment arm: $jd_{col} := 0.7 \cdot H_{col}$

Tension Force: $T_1 := \dfrac{Mp_1}{jd_{col}}$



Calculate joint shear stress demand ($\mathbf{v}_j$) and factored nominal joint shear strength ($\phi\mathbf{v}_n$)(ϕ=0.85). (See Joint Model flow chart)

Calculate corresponding vertical joint shear stress ($\mathbf{v}_j$) at maximum flexural strength of vertical members framing into joint

Joint shear force demand: $V_{joint} := T_1$ $\quad V_{joint} = 3958 \cdot kip$

vertical Joint cross-sectional area: $A_{joint} := 0.75 \cdot (H_{beam} \cdot H_{col})$ $\quad A_{joint} = 39\ ft^2$

Joint shear stress demand: $v_{jI} := \dfrac{V_{joint}}{A_{joint}}$ $\quad v_{jI} = 0.128 \cdot f_c \quad v_{jI} = 9.5 \cdot \sqrt{f_c \cdot psi}$

Categorize joint

- **Weak joint** -- Joints designed prior to the 1970's. Typically, these joints have minimal amounts, if any, of transverse reinforcement in the joint.
- **Moderate joint** – Joints designed between 1970 and 1990. These joints have a nominal amount of transverse reinforcement, enough to sustain concrete cracking without significant strength loss.
- **Intermediate joint** – Joints that have a nominal amount of transverse reinforcement, enough to sustain concrete cracking, but not enough to sustain yielding of the framing members. Bar yielding may be precluded by the lack of standard hooks, or by insufficient anchorage length for column bars passing through the joint.
- **Strong joint** -- Joints designed after 1990, containing significant amounts of horizontal and vertical reinforcement in the joint to enable proper confinement of the joint core and provide the necessary mechanisms for force transfer.

Calculate factored nominal joint shear strength, $\phi\mathbf{v}_n$ (ϕ=0.85)

Nominal Shear Strength	Weak Joint	Moderate Joint	Intermediate Joint	Strong Joint
$\mathbf{v}_n$	$v_n = 5\sqrt{f'_c}$	$v_n = 5\sqrt{f'_c}$	$v_n = 7.5\sqrt{f'_c}$	SDC limits

Joint shear strength: strength-reduction factor: $\phi := 0.85$

Weak & Moderate joint:

$vn_{weak} := 5 \cdot \sqrt{f_c \cdot psi}$ $\quad \phi \cdot vn_{weak} = 4.25 \cdot \sqrt{f_c \cdot psi}$

$vn_{mod} := 5 \cdot \sqrt{f_c \cdot psi}$ $\quad \phi \cdot vn_{mod} = 4.25 \cdot \sqrt{f_c \cdot psi}$

intermediate joint:

$vn_{int} := 7.5 \cdot \sqrt{f_c \cdot psi}$ $\quad \phi \cdot vn_{int} = 6.375 \cdot \sqrt{f_c \cdot psi}$

Strong joint, look at principal stress limits, per SDC:

(7.8) Principal compression $\quad p_c \le 0.25 \cdot f_c$

(7.9) Principal tension $\quad p_t \le 12 \cdot \sqrt{f_c \cdot psi}$

Principal Tensile stress:

$$p_t = \frac{f_h + f_v}{2} - \sqrt{\left(\frac{f_h - f_v}{2}\right)^2 + v_{jv}^2}$$

Principal Compressive stress:

$$p_c = \frac{f_h + f_v}{2} + \sqrt{\left(\frac{f_h - f_v}{2}\right)^2 + v_{jv}^2}$$

$$v_{jv} = \frac{T_c}{A_{jv}} \qquad A_{jv} = l_{ac} \cdot B_{cap} \qquad f_v = \frac{P_c}{A_{jh}} \qquad A_{jh} = \left(D_c + D_s\right) \cdot B_{cap} \qquad f_h = \frac{P_b}{B_{cap} \cdot D_s}$$

Where:

A_{jh} = The effective horizontal joint area
A_{jv} = The effective vertical joint area
B_{cap} = Bent cap width
D_c = Cross-sectional dimension of column in the direction of bending
D_s = Depth of superstructre at the bent cap
l_{ac} = Length of column reinforcement embedded into the bent cap
P_c = The column axial force including the effects of overturning
P_b = The beam axial force at the center of the joint including prestressing
T_c = The column tensile force defined as M_o^{col}/h, where h is the distance from c.g. of tensile force to c.g. of compressive force on the section, or alternatively T_c may be obtained from moment-curvature analysis of the cross section.

Converting the principal-stress limits to joint shear-stress limits (since the SDC do not use the strength reduction factor, to maintain consistency, v_n = SDC-limit value / ϕ).

max. allowable vertical shear stress based on principal tensile stress limits

$$v_{jT} \le \frac{1}{2} \cdot \sqrt{\left| -f_h^2 + 2 \cdot f_h \cdot f_v - f_v^2 + \left(f_h + f_v - 2 \cdot p_t\right)^2 \right|} \cdot \frac{1}{\phi}$$

max. allowable vertical shear stress based on principal compressive stress limits

$$v_{jC} \le \frac{1}{2} \cdot \sqrt{\left| -f_h^2 + 2 \cdot f_h \cdot f_v - f_v^2 + \left(2 \cdot p_c - f_h - f_v\right)^2 \right|} \cdot \frac{1}{\phi}$$

Applying to the frame:

$$B_{cap} := B_{beam} \qquad D_c := H_{col} \qquad D_s := H_{beam}$$

$l_{ac} := 0.90 \cdot H_{beam}$ Assume column longitudinal reinforcement is embedded across the entire beam depth (minus cover)

$P_c := \frac{Weight}{2}$ Overturning effects cannot be estimated at this time.

$P_b := 0 \cdot kip$ Assume no beam axial force in the bent

$$A_{jh} := \left(D_c + D_s\right) \cdot B_{cap} \qquad A_{jv} := l_{ac} \cdot B_{cap} \qquad f_v := \frac{P_c}{A_{jh}} \qquad f_h := \frac{P_b}{B_{cap} \cdot D_s}$$

SDC limits: $Pt_{max} := 12 \cdot \sqrt{f_c \cdot psi}$ $Pc_{max} := 0.25 \cdot f_c$ $Pc_{max} = 1375 \, psi$

max. allowable vertical shear stress based on principal tensile stress limits

$$v_{jmaxT} := \frac{1}{2}\cdot\sqrt{\left| -f_h^{\,2} + 2\cdot f_h\cdot f_v - f_v^{\,2} + \left(f_h + f_v - 2\cdot Pt_{max}\right)^2 \right|}\cdot\frac{1}{\phi}$$

max. allowable vertical shear stress based on principal compressive stress limits

$$v_{jmaxC} := \frac{1}{2}\cdot\sqrt{\left| -f_h^{\,2} + 2\cdot f_h\cdot f_v - f_v^{\,2} + \left(2\cdot Pc_{max} - f_h - f_v\right)^2 \right|}\cdot\frac{1}{\phi}$$

$v_{jmaxT} = 13.212\circ\sqrt{f_c\cdot psi}$ $\quad v_{jmaxC} = 20.917\circ\sqrt{f_c\cdot psi}$

<u>strong joint:</u> $vn_{strongI} := \min\left(\left[v_{jmaxT}\ \ v_{jmaxC}\right]\right)$ $\quad \phi\cdot vn_{strongI} = 11.23\circ\sqrt{f_c\cdot psi}$

$v_{jI} = 2.24\circ\phi\cdot vn_{weak}$

$v_{jI} = 2.24\circ\phi\cdot vn_{mod}$

$v_{jI} = 1.49\circ\phi\cdot vn_{int}$

$v_{jI} = 0.85\circ\phi\cdot vn_{strongI}$

Compare joint shear stress demand to factored strength.

$v_j < 0.5\ (\phi v_n)$ ① — Beam-column joint can be assumed rigid

$0.5\ (\phi v_n)\ \#\ v_j\ \#\ \phi v_n$ ② — Beam-column joint can be modeled as elastic member. Yielding of beam-column joints will occur without measurable strength loss.

$v_j\ \$\ \phi v_n$ ③ — Strength and stiffness degradation can be expected

$0.5\cdot\phi\cdot vn_{weak} = 2.125\circ\sqrt{f_c\cdot psi}$ $\quad \phi\cdot vn_{weak} = 4.25\circ\sqrt{f_c\cdot psi}$

$0.5\cdot\phi\cdot vn_{mod} = 2.125\circ\sqrt{f_c\cdot psi}$ $\quad \phi\cdot vn_{mod} = 4.25\circ\sqrt{f_c\cdot psi}$

$0.5\cdot\phi\cdot vn_{int} = 3.188\circ\sqrt{f_c\cdot psi}$ $\quad v_{jI} = 9.504\circ\sqrt{f_c\cdot psi}$ $\quad \phi\cdot vn_{int} = 6.375\circ\sqrt{f_c\cdot psi}$

$0.5\cdot\phi\cdot vn_{strongI} = 5.615\circ\sqrt{f_c\cdot psi}$ $\quad \phi\cdot vn_{strongI} = 11.23\circ\sqrt{f_c\cdot psi}$

Weak joint: $v_j > \phi\cdot v_n$ — strength and stiffness degradation can be expected

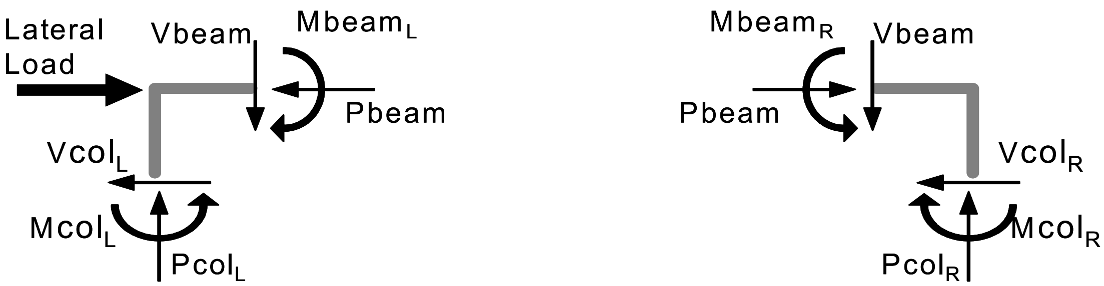
Moderate joint: $v_j > \phi\cdot v_n$ — strength and stiffness degradation can be expected

Intermediate joint: $v_j > \phi\cdot v_n$ — strength and stiffness degradation can be expected

Strong joint: $0.5\cdot\phi\cdot v_n < v_j < \phi\cdot v_n$ — Beam-column joint can be modeled as elastic member. Yielding of beam-column joint may occur without measurable strength loss.

Construct joint model (see Joint Model flow chart)

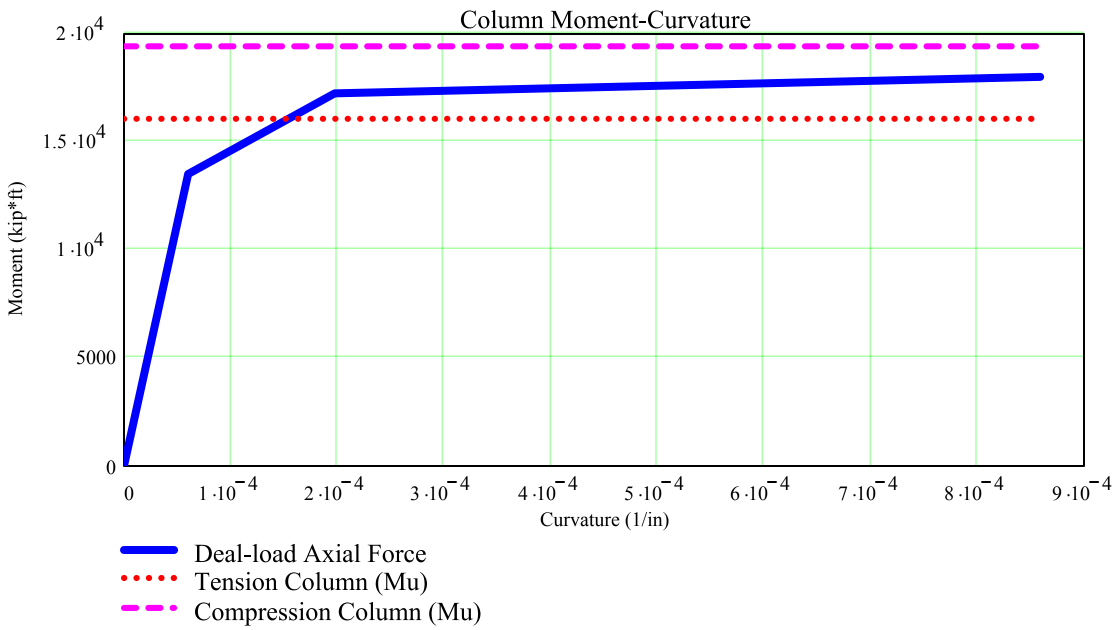
ALTERNATIVELY, the joint-boundary forces, and joint shear stress, can be obtained from a nonlinear pushover analysis of the frame to the prescribed limit state. Here, the limit state is defined by crushing of the concrete in the critical column section. A nonlinear pushover analysis was performed on a model of the bridge frame where nonlinear inelastic elements were used to represent the columns and an elastic element was used to represent the beam. The left and right columns of the bridge bent are referred to Tension Column and Compression Column, respectively, due to the effects of overturning. Both columns, however, are likely to be in compression, as the gravity axial loads exceed the overturning axial loads. The joint-boundary forces were obtained from this analysis at the column limit state:



Tension Column | **Compression Column**

$Pcol_L := 514 \cdot kip$	$Pbeam := 46.6 \cdot kip$	$Pcol_R := 2485.7 \cdot kip$
$Vcol_L := 446.3 \cdot kip$	$Vbeam := 985.7 \cdot kip$	$Vcol_R := 539.5 \cdot kip$
$Mbeam_L := 2.6278 \cdot 10^4 \cdot kip \cdot ft$		$Mbeam_R := 1.1991 \cdot 10^4 \cdot kip \cdot ft$
$Mcol_L := 1.6066 \cdot 10^4 \cdot kip \cdot ft$		$Mcol_R := 1.9421 \cdot 10^4 \cdot kip \cdot ft$

The column end moments can be compared to the ultimate moment of the column under dead-load axial force. The overturning tension and compression forces place the column-end moments above the DL ultimate moment for the case of the compression column and below the DL ultimate moment for the case of the tension column.

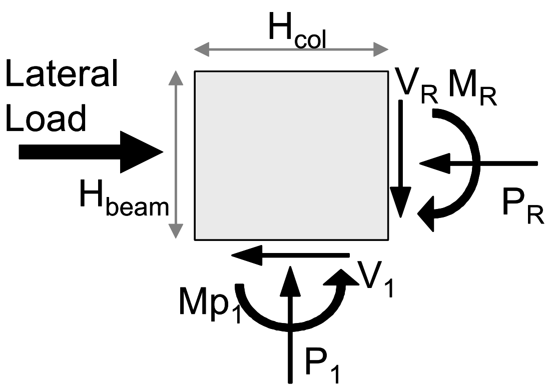


Analysis II

determine joint-boundary forces from Pushover analysis (tension column)

Even though the compression column is expected to result in the higher joint shear stresses, both joints will be evaluated.

Left-hand (Tension column) beam-column joint:



column:

$Mp_1 := Mcol_L$

$P_1 := Pcol_L$

$V_1 := Vcol_L$

$Mp_1 = 16066 \cdot kip \cdot ft$

$P_1 = 514 \cdot kip$

$V_1 = 446.3 \cdot kip$

beam:

$M_R := Mbeam_L$

$P_R := Pbeam$

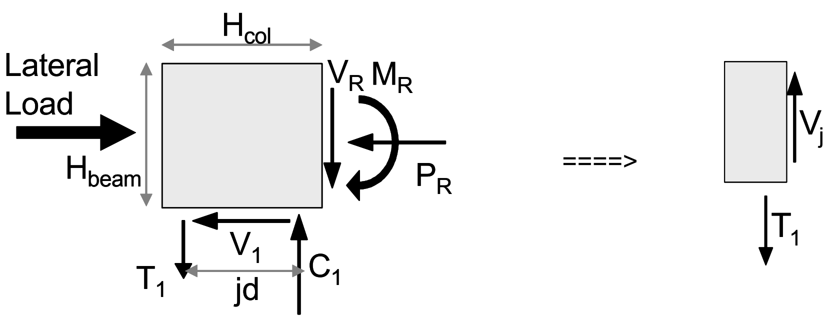
$V_R := Vbeam$

$M_R = 26278 \cdot kip \cdot ft$

$P_R = 46.6 \cdot kip$

$V_R = 986 \cdot kip$

In converting the column moment into a couple we realize that the only item of interest is actually T_1, the tension component of the couple



$jd_{col} := 0.7 \cdot H_{col}$

$T_1 := \dfrac{Mp_1}{jd_{col}}$

Cross-sectional area of joint:

$A_{joint} = 39\ ft^2$

Joint shear force:

$V_{joint} := T_1$

$V_{joint} = 3531 \cdot kip$

Joint shear stress:

$v_{jII} := \dfrac{V_{joint}}{A_{joint}}$

$v_{jII} = 8.5 \cdot \sqrt{f_c \cdot psi}$

from a section analysis we had:

$v_{jI} = 9.5 \cdot \sqrt{f_c \cdot psi}$

the section-analysis case is more conservative.

Comparing the joint shear-stress demand to the factored strengths:

$v_{jII} = 1.99 \cdot \phi \cdot vn_{weak}$

$v_{jII} = 1.99 \cdot \phi \cdot vn_{mod}$

$v_{jII} = 1.33 \cdot \phi \cdot vn_{int}$

In this case, we can incorporate the actual value for the column and beam axial forces in determining the strength of the strong joint, based on the principal-stress ratios:

Converting the principal-stress limits to joint shear-stress limits:

max. allowable vertical shear stress based on principal tensile stress limits

$$v_{jT} \le \frac{1}{2} \cdot \sqrt{\left| -f_h^2 + 2 \cdot f_h \cdot f_v - f_v^2 + \left(f_h + f_v - 2 \cdot p_t\right)^2 \right|} \cdot \frac{1}{\phi}$$

max. allowable vertical shear stress based on principal compressive stress limits

$$v_{jC} \le \frac{1}{2} \cdot \sqrt{\left| -f_h^2 + 2 \cdot f_h \cdot f_v - f_v^2 + \left(2 \cdot p_c - f_h - f_v\right)^2 \right|} \cdot \frac{1}{\phi}$$

Applying to the frame:

$B_{cap} := B_{beam}$ $\quad D_c := H_{col}$ $\quad D_s := H_{beam}$

$l_{ac} := 0.90 \cdot H_{beam}$ Assume column longitudinal reinforcement is embedded across the entire beam depth (minus cover)

$P_c := Pcol_L$ Overturning effects cannot be estimated at this time.

$P_b := Pbeam$ Assume no beam axial force in the bent

$A_{jh} := \left(D_c + D_s\right) \cdot B_{cap}$ $\quad A_{jv} := l_{ac} \cdot B_{cap}$ $\quad f_v := \dfrac{P_c}{A_{jh}}$ $\quad f_h := \dfrac{P_b}{B_{cap} \cdot D_s}$

SDC limits: $Pt_{max} := 12 \cdot \sqrt{f_c \cdot psi}$ $\quad Pc_{max} := 0.25 \cdot f_c$ $\quad Pc_{max} = 1375 \cdot psi$

max. allowable vertical shear stress based on principal tensile stress limits

$$v_{jmaxT} := \frac{1}{2} \cdot \sqrt{\left| -f_h^2 + 2 \cdot f_h \cdot f_v - f_v^2 + \left(f_h + f_v - 2 \cdot Pt_{max}\right)^2 \right|} \cdot \frac{1}{\phi}$$

max. allowable vertical shear stress based on principal compressive stress limits

$$v_{jmaxC} := \frac{1}{2} \cdot \sqrt{\left| -f_h^2 + 2 \cdot f_h \cdot f_v - f_v^2 + \left(2 \cdot Pc_{max} - f_h - f_v\right)^2 \right|} \cdot \frac{1}{\phi}$$

$$v_{jmaxT} = 13.766 \cdot \sqrt{f_c \cdot psi} \qquad v_{jmaxC} = 21.461 \cdot \sqrt{f_c \cdot psi}$$

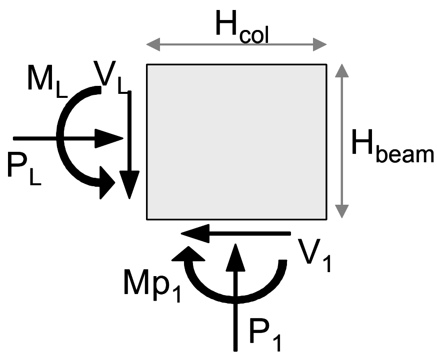
<u>strong joint:</u> $vn_{strongII} := \min\left(\left[v_{jmaxT} \;\; v_{jmaxC}\right]\right)$ $\quad \phi \cdot vn_{strongII} = 11.701 \cdot \sqrt{f_c \cdot psi}$

Therefore: $v_{jII} = 0.72 \cdot \phi \cdot vn_{strongII}$

Analysis II

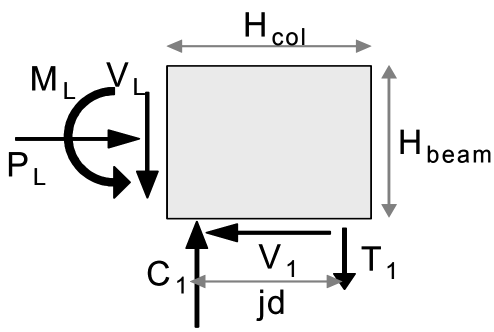
determine joint-boundary forces from Pushover analysis (tension column)

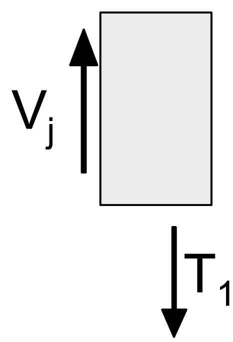
Right-hand (Compression column) beam-column joint:



column:	beam:
$Mp_1 := Mcol_R$	$M_L := Mbeam_R$
$P_1 := Pcol_R$	$P_L := Pbeam$
$V_1 := Vcol_R$	$V_L := Vbeam$
$Mp_1 = 19421 \circ kip \cdot ft$	$M_L = 1.199 \cdot 10^4 \circ kip \cdot ft$
$P_1 = 2.486 \cdot 10^3 \circ kip$	$P_L = 46.6 \circ kip$
$V_1 = 539.5 \circ kip$	$V_L = 986 \circ kip$

In converting the column moment into a couple we realize that the only item of interest is actually T_1, the tension component of the couple





$$jd_{col} := 0.7 \cdot H_{col}$$

$$T_1 := \frac{Mp_1}{jd_{col}}$$

Cross-sectional area of joint: $A_{joint} = 39\ ft^2$

Joint shear force: $V_{joint} := T_1$ $\quad V_{joint} = 4268 \circ kip$

Joint shear stress:

$$v_{jIII} := \frac{V_{joint}}{A_{joint}}$$

$$v_{jIII} = 10.2 \circ \sqrt{f_c \cdot psi}$$ <<<<<

from a simple section analysis we had: $v_{jI} = 9.5 \circ \sqrt{f_c \cdot psi}$

The simple analysis yielded a lower joint shear stress than the nonlinear pushover analysis, as expected. The error, however, is within reasonable bounds (5%). It is, however, recommended that the nonlinear pushover analysis be used in determining joint-boundary forces.

Comparing the joint shear-stress demand to the factored strengths:

$$v_{jIII} = 2.41 \circ \phi \cdot vn_{weak}$$

$$v_{jIII} = 2.41 \circ \phi \cdot vn_{mod}$$

$$v_{jIII} = 1.61 \circ \phi \cdot vn_{int}$$

In this case, we can incorporate the actual value for the column and beam axial forces in determining the strength of the strong joint, based on the principal-stress ratios:

Converting the principal-stress limits to joint shear-stress limits:

max. allowable vertical shear stress based on principal tensile stress limits
$$v_{jT} \le \frac{1}{2}\cdot\sqrt{\left| -f_h^2 + 2\cdot f_h\cdot f_v - f_v^2 + \left(f_h + f_v - 2\cdot p_t\right)^2 \right|}\cdot\frac{1}{\phi}$$

max. allowable vertical shear stress based on principal compressive stress limits
$$v_{jC} \le \frac{1}{2}\cdot\sqrt{\left| -f_h^2 + 2\cdot f_h\cdot f_v - f_v^2 + \left(2\cdot p_c - f_h - f_v\right)^2 \right|}\cdot\frac{1}{\phi}$$

Applying to the frame:

$B_{cap} := B_{beam}$ $\quad D_c := H_{col}$ $\quad D_s := H_{beam}$

$l_{ac} := 0.90\cdot H_{beam}$ Assume column longitudinal reinforcement is embedded across the entire beam depth (minus cover)

$P_c := Pcol_R$ Overturning effects cannot be estimated at this time.

$P_b := Pbeam$ Assume no beam axial force in the bent

$A_{jh} := \left(D_c + D_s\right)\cdot B_{cap}$ $\quad A_{jv} := l_{ac}\cdot B_{cap}$ $\quad f_v := \dfrac{P_c}{A_{jh}}$ $\quad f_h := \dfrac{P_b}{B_{cap}\cdot D_s}$

SDC limits: $Pt_{max} := 12\cdot\sqrt{f_c\cdot psi}$ $\quad Pc_{max} := 0.25\cdot f_c$ $\quad Pc_{max} = 1375\,psi$

max. allowable vertical shear stress based on principal tensile stress limits
$$v_{jmaxT} := \frac{1}{2}\cdot\sqrt{\left| -f_h^2 + 2\cdot f_h\cdot f_v - f_v^2 + \left(f_h + f_v - 2\cdot Pt_{max}\right)^2 \right|}\cdot\frac{1}{\phi}$$

max. allowable vertical shear stress based on principal compressive stress limits
$$v_{jmaxC} := \frac{1}{2}\cdot\sqrt{\left| -f_h^2 + 2\cdot f_h\cdot f_v - f_v^2 + \left(2\cdot Pc_{max} - f_h - f_v\right)^2 \right|}\cdot\frac{1}{\phi}$$

$v_{jmaxT} = 12.537\,\sqrt{f_c\cdot psi}$ $\quad v_{jmaxC} = 20.262\,\sqrt{f_c\cdot psi}$

<u>strong joint:</u> $vn_{strongIII} := \min\left(\left[v_{jmaxT} \;\; v_{jmaxC}\right]\right)$ $\quad \phi\cdot vn_{strongIII} = 10.657\,\sqrt{f_c\cdot psi}$

Therefore: $v_{jIII} = 0.96\,\phi\cdot vn_{strongIII}$

Factor for stong-joint model: THIS CASE CONTROLS

$$SDC := \min\left(\begin{bmatrix} \frac{1}{2}\cdot\sqrt{\left| -f_h^2 + 2\cdot f_h\cdot f_v - f_v^2 + \left(f_h + f_v - 2\cdot Pt_{max}\right)^2 \right|} \\ \frac{1}{2}\cdot\sqrt{\left| -f_h^2 + 2\cdot f_h\cdot f_v - f_v^2 + \left(2\cdot Pc_{max} - f_h - f_v\right)^2 \right|} \end{bmatrix}\right)\cdot\frac{1}{\sqrt{f_c\cdot psi}}$$

$SDC = 10.7$

The results from the nonlinear analysis can be used, yielding the same conclusions as the section analysis:

$v_j < 0.5\ (\phi v_n)$ (1)

Beam-column joint can be assumed rigid

$0.5\ (\phi v_n)\ \#\ v_j\ \#\ \phi v_n$ (2)

Beam-column joint can be modeled as elastic member. Yielding of beam-column joints will occur without measurable strength loss.

$v_j\ \$\ \phi v_n$ (3)

Strength and stiffness degradation can be expected

$$0.5\cdot\phi\cdot vn_{weak} = 2.125\circ\sqrt{f_c\cdot psi}$$

$$0.5\cdot\phi\cdot vn_{mod} = 2.125\circ\sqrt{f_c\cdot psi}$$

$$0.5\cdot\phi\cdot vn_{int} = 3.188\circ\sqrt{f_c\cdot psi}$$

$$0.5\cdot\phi\cdot vn_{strongIII} = 5.328\circ\sqrt{f_c\cdot psi}$$

$$v_{jIII} = 10.248\circ\sqrt{f_c\cdot psi}$$

$$\phi\cdot vn_{weak} = 4.25\circ\sqrt{f_c\cdot psi}$$

$$\phi\cdot vn_{mod} = 4.25\circ\sqrt{f_c\cdot psi}$$

$$\phi\cdot vn_{int} = 6.375\circ\sqrt{f_c\cdot psi}$$

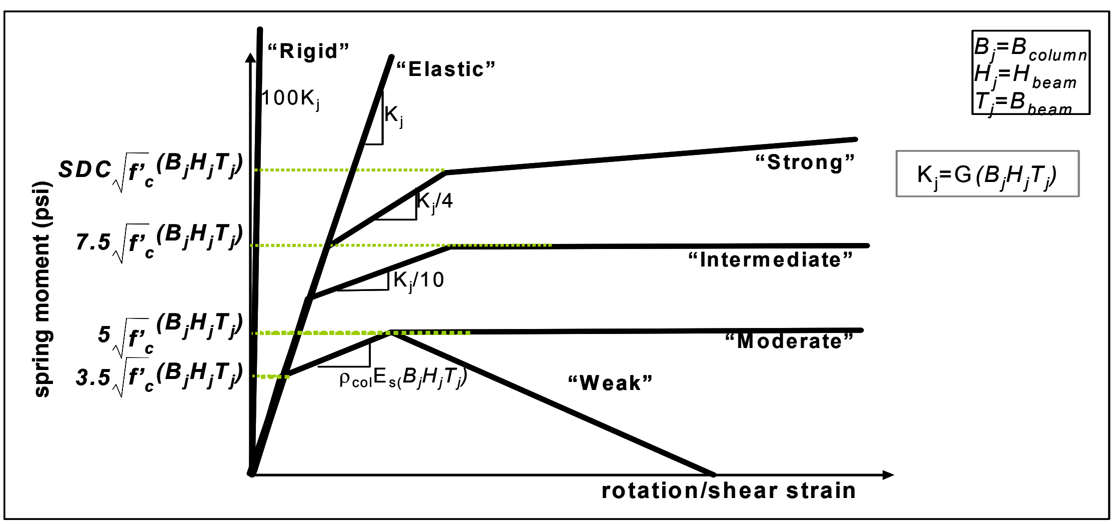
$$\phi\cdot vn_{strongIII} = 10.657\circ\sqrt{f_c\cdot psi}$$

Weak & moderate joint: $v_j > \phi\cdot v_n$ strength and stiffness degradation can be expected

Intermediate joint: $v_j > \phi\cdot v_n$ strength and stiffness degradation can be expected

Strong joint: $0.5\cdot\phi\cdot v_n < v_j < \phi\cdot v_n$ Yielding of beam-column joint will occur without strength loss.

Construct joint model (see Joint Model flow chart)



Joint geometry: $B_j := H_{col}$ $H_j := H_{beam}$ $T_j := B_{beam}$

Elastic stiffness of joint spring: $K_j := G_c \cdot (B_j \cdot H_j \cdot T_j)$ $K_j = 8.573 \cdot 10^7 \circ \frac{kip \cdot ft}{rad}$

Weak Joint Model:

Cracking strength: $Mcr_w := 3.5 \cdot \sqrt{f_c \cdot psi} \cdot (B_j \cdot H_j \cdot T_j)$ $Mcr_w = 12634 \circ kip \cdot ft$

Initial stiffness: $K1_w := K_j$ $K1_w = 8.573 \cdot 10^7 \circ \frac{kip \cdot ft}{rad}$

Rotation at cracking: $\theta cr_w := \frac{Mcr_w}{K1_w}$ $\theta cr_w = 1.474 \cdot 10^{-4} \circ rad$

Post-cracking stiffness: $K2_w := \rho_{col} \cdot E_s \cdot (B_j \cdot H_j \cdot T_j)$ $K2_w = 0.288 \circ K_j$

Yield strength: $My_w := 5 \cdot \sqrt{f_c \cdot psi} \cdot (B_j \cdot H_j \cdot T_j)$ $My_w = 18048 \circ kip \cdot ft$

Rotation at yield: $\theta y_w := \theta cr_w + \frac{My_w - Mcr_w}{K2_w}$ $\theta y_w = 3.666 \cdot 10^{-4} \circ rad$

Ultimate strength: $Mu_w := 0 \cdot \sqrt{f_c \cdot psi} \cdot (B_j \cdot H_j \cdot T_j)$ $Mu_w = 0 \circ kip \cdot ft$

Rotation at ultimate: $\theta u_w := 0.01$

Post-yield stiffness: $K3_w := \frac{Mu_w - My_w}{\theta u_w - \theta y_w}$ $K3_w = -0.022 \circ K_j$

vectorize: $\Theta j_w := \begin{bmatrix} 0 \cdot rad & \theta cr_w & \theta y_w & \theta u_w \end{bmatrix}^T$ $Mj_w := \begin{bmatrix} 0 \cdot \frac{kip \cdot ft}{rad} & Mcr_w & My_w & Mu_w \end{bmatrix}^T$

Moderate Joint Model: (this joint model has the same pre-yield characteristics as the weak model. There is, however, a nominal amount of reinforcement in the joint to prevent immediate strength loss) In this example, the joint is actually able to sustain the strength.

Cracking strength: $Mcr_m := Mcr_w$ $Mcr_m = 12634 \circ kip \cdot ft$

Initial stiffness: $K1_m := K1_w$ $K1_m = 8.573 \cdot 10^7 \circ \frac{kip \cdot ft}{rad}$

Rotation at cracking: $\theta cr_m := \theta cr_w$ $\theta cr_m = 1.474 \cdot 10^{-4} \circ rad$

Post-cracking stiffness: $K2_m := K2_w$ $K2_m = 0.288 \circ K_j$

Yield strength: $My_m := My_w$ $My_m = 18048 \circ kip \cdot ft$

Rotation at yield: $\theta y_m := \theta y_w$ $\theta y_m = 3.666 \cdot 10^{-4} \circ rad$

Ultimate strength: $Mu_m := 1.000000001 \cdot My_m$ $Mu_m = 1.805 \cdot 10^4 \circ kip \cdot ft$

Rotation at ultimate: $\theta u_m := 0.01$

Post-yield stiffness: $K3_m := \frac{Mu_m - My_m}{\theta u_m - \theta y_m}$ $K3_m = 2.185 \cdot 10^{-11} \circ K_j$

vectorize: $\Theta j_m := \begin{bmatrix} 0 \cdot rad & \theta cr_m & \theta y_m & \theta u_m \end{bmatrix}^T$ $\quad Mj_m := \begin{bmatrix} 0 \cdot \frac{kip \cdot ft}{rad} & Mcr_m & My_m & Mu_m \end{bmatrix}^T$

Intermediate Joint Model:

Cracking strength: $Mcr_i := 5 \cdot \sqrt{f_c \cdot psi} \cdot (B_j \cdot H_j \cdot T_j)$ $\quad Mcr_i = 18048 \cdot kip \cdot ft$

Initial stiffness: $K1_i := K_j$ $\quad K1_i = 8.573 \cdot 10^7 \cdot \frac{kip \cdot ft}{rad}$

Rotation at cracking: $\theta cr_i := \frac{Mcr_i}{K1_i}$ $\quad \theta cr_i = 2.105 \cdot 10^{-4} \cdot rad$

Post-cracking stiffness: $K2_i := \frac{K1_i}{10}$ $\quad K2_i = 0.1 \cdot K_j$

Yield strength: $My_i := 7.5 \cdot \sqrt{f_c \cdot psi} \cdot (B_j \cdot H_j \cdot T_j)$ $\quad My_i = 27072 \cdot kip \cdot ft$

Rotation at yield: $\theta y_i := \theta cr_i + \frac{My_i - Mcr_i}{K2_i}$ $\quad \theta y_i = 1.263 \cdot 10^{-3} \cdot rad$

The ultimate strength of the intermediate joint needs to be determined by the designer. The case considered in this example is an intermediate joint with suffient confinement to sustain deformations beyond yield without strength loss:

Ultimate strength: $Mu_i := 1.001 \cdot My_i$ $\quad Mu_i = 27099 \cdot kip \cdot ft$

Rotation at ultimate: $\theta u_i := 0.1$

Post-yield stiffness: $K3_i := \frac{Mu_i - My_i}{\theta u_i - \theta y_i}$ $\quad K3_i = 3.198 \cdot 10^{-6} \cdot K_j$

vectorize: $\Theta j_i := \begin{bmatrix} 0 \cdot rad & \theta cr_i & \theta y_i & \theta u_i \end{bmatrix}^T$ $\quad Mj_i := \begin{bmatrix} 0 \cdot \frac{kip \cdot ft}{rad} & Mcr_i & My_i & Mu_i \end{bmatrix}^T$

Strong Joint Model: $SDC = 10.7$

Cracking strength: $Mcr_s := 7.5 \cdot \sqrt{f_c \cdot psi} \cdot (B_j \cdot H_j \cdot T_j)$ $\quad Mcr_s = 27072 \cdot kip \cdot ft$

Initial stiffness: $K1_s := K_j$ $\quad K1_s = 8.573 \cdot 10^7 \cdot \frac{kip \cdot ft}{rad}$

Rotation at cracking: $\theta cr_s := \frac{Mcr_s}{K1_s}$ $\quad \theta cr_s = 3.158 \cdot 10^{-4} \cdot rad$

Post-cracking stiffness: $K2_s := \frac{K1_s}{10}$ $\quad K2_s = 0.1 \cdot K_j$

Yield strength: $My_s := SDC \cdot \sqrt{f_c \cdot psi} \cdot (B_j \cdot H_j \cdot T_j)$ $\quad My_s = 38467 \cdot kip \cdot ft$

Rotation at yield: $\theta y_s := \theta cr_s + \frac{My_s - Mcr_s}{K2_s}$ $\quad \theta y_s = 1.645 \cdot 10^{-3} \cdot rad$

The ultimate strength of the strong joint needs to be determined by the designer. The case considered in this example is an intermediate joint with suffient confinement to sustain deformations well beyond yield with significant strength gain:

Ultimate strength: $Mu_s := 1.25 \cdot My_s$ $\quad Mu_s = 48083 \cdot kip \cdot ft$

Rotation at ultimate: $\theta u_s := 0.1$

Post-yield stiffness: $K3_s := \dfrac{Mu_s - My_s}{\theta u_s - \theta y_s}$ $\quad K3_s = 1.141 \cdot 10^{-3} \cdot K_j$

vectorize: $\Theta j_s := \begin{bmatrix} 0 \cdot rad & \theta cr_s & \theta y_s & \theta u_s \end{bmatrix}^T$ $\quad Mj_s := \begin{bmatrix} 0 \cdot \dfrac{kip \cdot ft}{rad} & Mcr_s & My_s & Mu_s \end{bmatrix}^T$

Elastic Joint Model:

Cracking strength: $Mcr_e := 7.5 \cdot \sqrt{f_c \cdot psi} \cdot \left(B_j \cdot H_j \cdot T_j\right)$ $\quad Mcr_e = 27072 \cdot kip \cdot ft$

Initial stiffness: $K1_e := K_j$ $\quad K1_e = 8.573 \cdot 10^{7} \cdot \dfrac{kip \cdot ft}{rad}$

Rotation at cracking: $\theta cr_e := \dfrac{Mcr_e}{K1_e}$ $\quad \theta cr_e = 3.158 \cdot 10^{-4} \cdot rad$

Post-cracking stiffness: $K2_e := K1_e$ $\quad K2_e = 1 \cdot K_j$

Yield strength: $My_e := 15 \cdot \sqrt{f_c \cdot psi} \cdot \left(B_j \cdot H_j \cdot T_j\right)$ $\quad My_e = 54144 \cdot kip \cdot ft$

Rotation at yield: $\theta y_e := \theta cr_e + \dfrac{My_e - Mcr_e}{K2_e}$ $\quad \theta y_e = 6.316 \cdot 10^{-4} \cdot rad$

Ultimate strength: $Mu_e := 1.25 \cdot My_e$ $\quad Mu_e = 67680 \cdot kip \cdot ft$

Post-yield stiffness: $K3_e := K1_e$ $\quad K3_e = 1 \cdot K_j$

Rotation at ultimate: $\theta u_e := \dfrac{Mu_e}{K3_e}$

vectorize: $\Theta j_e := \begin{bmatrix} 0 \cdot rad & \theta cr_e & \theta y_e & \theta u_e \end{bmatrix}^T$ $\quad Mj_e := \begin{bmatrix} 0 \cdot \dfrac{kip \cdot ft}{rad} & Mcr_e & My_e & Mu_e \end{bmatrix}^T$

Rigid Joint Model:

Cracking strength: $Mcr_r := 7.5 \cdot \sqrt{f_c \cdot psi} \cdot \left(B_j \cdot H_j \cdot T_j\right)$ $\quad Mcr_r = 27072 \cdot kip \cdot ft$

Initial stiffness: $K1_r := 100 \cdot K_j$ $\quad K1_r = 8.573 \cdot 10^{9} \cdot \dfrac{kip \cdot ft}{rad}$

Rotation at cracking: $\theta cr_r := \dfrac{Mcr_r}{K1_r}$ $\quad \theta cr_r = 3.158 \cdot 10^{-6} \cdot rad$

Post-cracking stiffness: $K2_r := K1_r$ $\quad K2_r = 100 \cdot K_j$

Yield strength: $My_r := 15 \cdot \sqrt{f_c \cdot psi} \cdot \left(B_j \cdot H_j \cdot T_j\right)$ $\quad My_r = 54144 \cdot kip \cdot ft$

Rotation at yield: $\theta y_r := \theta cr_r + \dfrac{My_r - Mcr_r}{K2_r}$ $\quad \theta y_r = 6.316 \cdot 10^{-6} \cdot rad$

Ultimate strength: $Mu_r := 1.25 \cdot My_r$ $Mu_r = 67680 \cdot kip \cdot ft$

Post-yield stiffness: $K3_r := K1_r$ $K3_r = 100 \cdot K_j$

Rotation at ultimate: $\theta u_r := \dfrac{Mu_r}{K3_r}$

vectorize: $\Theta j_r := \left[0 \cdot rad \;\; \theta cr_r \;\; \theta y_r \;\; \theta u_r\right]^T$ $Mj_r := \left[0 \cdot \dfrac{kip \cdot ft}{rad} \;\; Mcr_r \;\; My_r \;\; Mu_r\right]^T$

Weak Joint Model:

$\Theta j_w^T = (0 \;\; 0.00015 \;\; 0.00037 \;\; 0.01) \cdot rad$

$Mj_w^T = (0 \;\; 12634 \;\; 18048 \;\; 0) \cdot kip \cdot ft$

Moderate Joint Model:

$\Theta j_m^T = (0 \;\; 0.00015 \;\; 0.00037 \;\; 0.01) \cdot rad$

$Mj_m^T = (0 \;\; 12634 \;\; 18048 \;\; 18048) \cdot kip \cdot ft$

Intermediate Joint Model:

$\Theta j_i^T = (0 \;\; 0.00021 \;\; 0.00126 \;\; 0.1) \cdot rad$

$Mj_i^T = (0 \;\; 18048 \;\; 27072 \;\; 27099) \cdot kip \cdot ft$

Strong Joint Model:

$\Theta j_s^T = \left[0 \;\; 3.158 \cdot 10^{-4} \;\; 1.645 \cdot 10^{-3} \;\; 0.1\right] \cdot rad$

$Mj_s^T = (0 \;\; 27072 \;\; 38467 \;\; 48083) \cdot kip \cdot ft$

Elastic Joint Model:

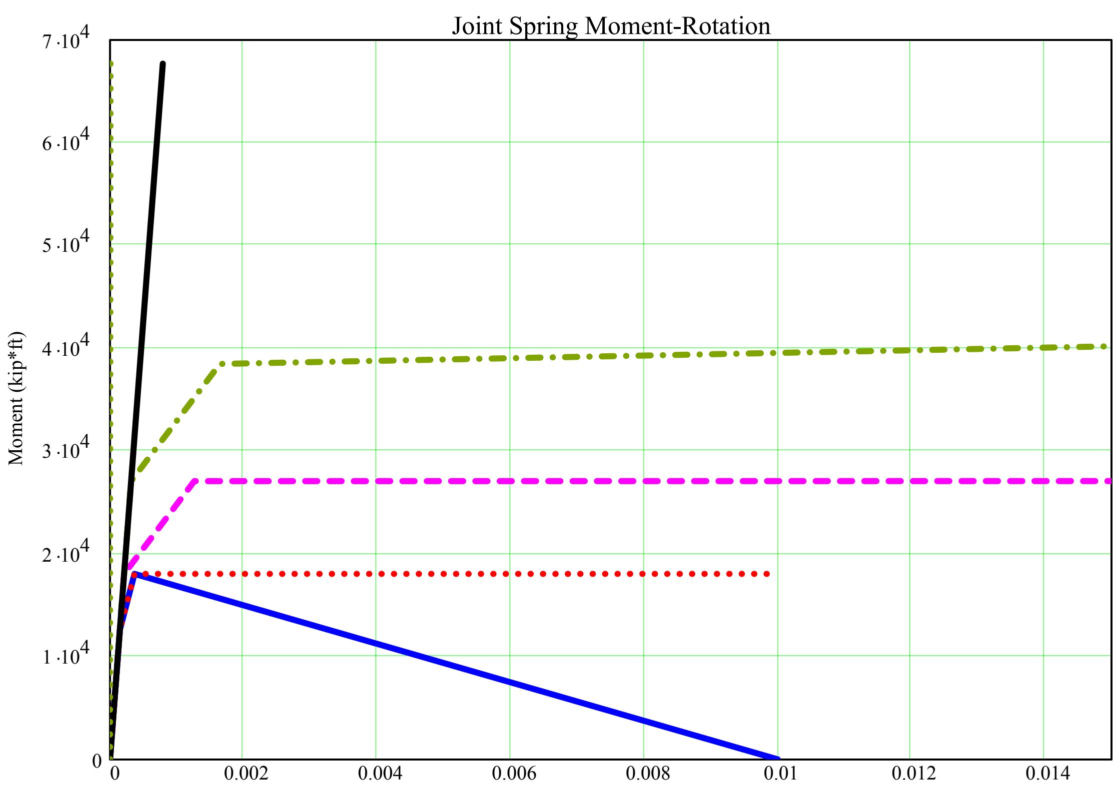
$\Theta j_e^T = (0 \;\; 0.00032 \;\; 0.00063 \;\; 0.00079) \cdot rad$

$Mj_e^T = (0 \;\; 27072 \;\; 54144 \;\; 67680) \cdot kip \cdot ft$

Rigid Joint Model:

$\Theta j_r^T = \left[0 \;\; 3.158 \cdot 10^{-6} \;\; 6.316 \cdot 10^{-6} \;\; 7.895 \cdot 10^{-6}\right] \cdot rad$

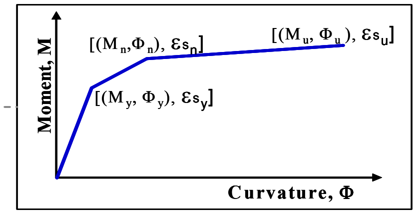
$Mj_r^T = (0 \;\; 27072 \;\; 54144 \;\; 67680) \cdot kip \cdot ft$



Curvature (1/in)

Construct hinge model (see Hinge Model flow chart)

Perform Moment-Curvature analysis of column section. Calculate moment-curvature data: M_y, M_n, M_u, and corresponding steel strain (εs_y, εs_n, εs_u). Save section properties: column diameter (H_c), longitudinal-bar diameter (d_b)



The first part of this task was performed in the design process:

$$\Phi ynu_{col} = \begin{bmatrix} 0 \\ 6.012 \cdot 10^{-5} \\ 1.975 \cdot 10^{-4} \\ 8.589 \cdot 10^{-4} \end{bmatrix} \frac{1}{in} \qquad Mynu_{col} = \begin{bmatrix} 0 \\ 13511 \\ 17248 \\ 18010 \end{bmatrix} kip \cdot ft$$

The steel strains at the moment-curvature points need to be extracted from the moment-curvature analysis. They can, however, be determined from the data:

steel strain at section yield strength $\qquad \varepsilon s_y := \frac{f_y}{E_s}$

assume the column core diameter is 90% of the column diameter: $\qquad H_{core} := 0.9 \cdot H_{col}$

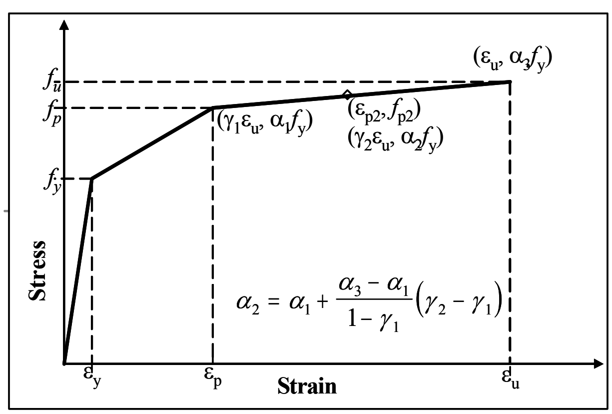
The nominal strength of the column is defined by the concrete strain: $\qquad \varepsilon c_n := 0.003$

Assuming a linear curvature distribution, steel strain at nominal section flexural strength: $\qquad \varepsilon s_n := \phi n_{col} \cdot H_{core} - \varepsilon c_n \qquad \varepsilon s_n = 0.011$

The ultimate strength of the column is defined by the concrete strain: $\qquad \varepsilon c_u := 0.004 + 1.4 \cdot \left(\rho_{col} \cdot f_y\right) \cdot \frac{\varepsilon_u}{f_c} \qquad \varepsilon c_u = 0.034$

Assuming a linear curvature distribution, steel strain at ultimate section flexural strength: $\qquad \varepsilon s_u := \phi u_{col} \cdot H_{core} - \varepsilon c_u \qquad \varepsilon s_u = 0.026$

Determine simplified steel and concrete material model: f_y, ε_y, ε_u, α_1, α_2, α_3, γ_1, γ_2, f_c'



The following values are recommended for nominal material properties, based on an approximation of the SBD steel model:

$\alpha_1 := 1.32$ = ratio of steel plastic stress (initiation of strain hardening) to yield stress ($\alpha_1 = f_p/f_y$) $f_p := \alpha_1 \cdot f_y$

$\alpha_3 := 1.4$ = ratio of steel ultimate stress to yield stress ($\alpha_2 = f_u/f_y$) $f_u := \alpha_3 \cdot f_y$

$\gamma_1 := 0.5$ = ratio of steel plastic strain to ultimate strain ($\gamma_1 = \varepsilon_p/\varepsilon_u$) $\varepsilon_p := \gamma_1 \cdot \varepsilon_u$

$\gamma_2 := 0.75$ = ratio of secondary steel plastic strain (an intermediate point between ultimate and ε_p) to ultimate strain ($\gamma_2 = \varepsilon_p 2/\varepsilon_u$) $\varepsilon_{p2} := \gamma_2 \cdot \varepsilon_u$

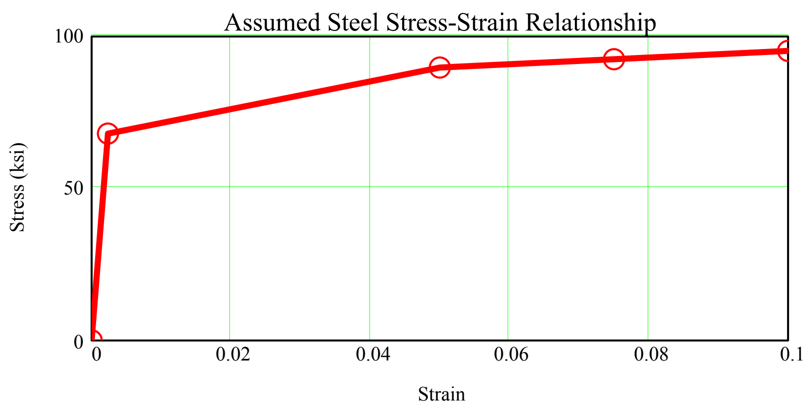
$\varepsilon_p = 0.05$

$\varepsilon_y := \dfrac{f_y}{E_s}$ = steel yield strain

$\alpha_2 := \alpha_1 + \dfrac{\alpha_3 - \alpha_1}{1 - \gamma_1} \cdot (\gamma_2 - \gamma_1)$ = ratio of secondary steel plastic stress (an intermediate point between ultimate and ε_p) $f_{p2} := \alpha_2 \cdot f_y$

$\alpha_2 = 1.36$

$\varepsilon_s := \begin{bmatrix} 0 & \varepsilon_y & \varepsilon_p & \varepsilon_{p2} & \varepsilon_u \end{bmatrix}^T$ $F_s := \begin{bmatrix} 0 \cdot psi & f_y & f_p & f_{p2} & f_u \end{bmatrix}^T$



Select Bond-Stress Model

"Strong" bond model :	"Intermediate" bond model:	"Weak" bond model:	bar stress
$u_e = 30\sqrt{f'_c}$	$u_e = 30\sqrt{f'_c}$	$u_e = 12\sqrt{f'_c}$	pre-yield
$u_p = 30\sqrt{f'_c}$	$u_p = 15\sqrt{f'_c}$	$u_p = 6\sqrt{f'_c}$	post-yield

Weak bond model: pre-yield bond stress: $u_{ew} := 12 \cdot \sqrt{f_c \cdot psi}$

post-yield bond stress: $u_{pw} := 6 \cdot \sqrt{f_c \cdot psi}$

Intermediate bond model: pre-yield bond stress: $u_{ei} := 30 \cdot \sqrt{f_c \cdot psi}$

post-yield bond stress: $u_{pi} := 15 \cdot \sqrt{f_c \cdot psi}$

Strong bond model: pre-yield bond stress: $u_{es} := 30 \cdot \sqrt{f_c \cdot psi}$

post-yield bond stress: $u_{ps} := 30 \cdot \sqrt{f_c \cdot psi}$

Determine rotation vs. steel-strain relationship

$$\begin{bmatrix} \varepsilon_y, \left(\theta_y = \frac{1}{4}\cdot\frac{d_b}{H_c}\cdot\varepsilon_y\cdot\frac{f_y}{u_e}\right) \\ (\gamma_1\cdot\varepsilon_u), \left[\theta_p = \theta_y + \frac{1}{4}\cdot\frac{d_b}{H_c}\cdot(\varepsilon_y+\gamma_1\cdot\varepsilon_u)\cdot(\alpha_1-1)\cdot\frac{f_y}{u_p}\right] \\ (\gamma_2\cdot\varepsilon_u), \left[\theta_{p1} = \theta_y + \frac{1}{4}\cdot\frac{d_b}{H_c}\cdot\left[(\varepsilon_y+\gamma_1\cdot\varepsilon_u)\cdot(\alpha_1-1)+\varepsilon_u\cdot(\gamma_1+\gamma_2)\cdot(\alpha_2-\alpha_1)\right]\cdot\frac{f_y}{u_p}\right] \\ \varepsilon_u, \left[\theta_u = \theta_y + \frac{1}{4}\cdot\frac{d_b}{H_c}\cdot\left[(\varepsilon_y+\gamma_1\cdot\varepsilon_u)\cdot(\alpha_1-1)+\varepsilon_u\cdot(1+\gamma_1)\cdot(\alpha_3-\alpha_1)\right]\cdot\frac{f_y}{u_p}\right] \end{bmatrix}$$

Weak bond model:

$$\theta_{yw} := \frac{1}{4}\cdot\frac{d_b}{H_{col}}\cdot\varepsilon_y\cdot\frac{f_y}{u_{ew}}$$

$\theta_{yw} = 0.00097\,\text{rad}$

$$\theta_{pw} := \theta_{yw} + \frac{1}{4}\cdot\frac{d_b}{H_{col}}\cdot(\varepsilon_y+\gamma_1\cdot\varepsilon_u)\cdot(\alpha_1-1)\cdot\frac{f_y}{u_{pw}}$$

$\theta_{pw} = 0.015\,\text{rad}$

$$\theta_{p1w} := \theta_{yw} + \frac{1}{4}\cdot\frac{d_b}{H_{col}}\cdot\left[(\varepsilon_y+\gamma_1\cdot\varepsilon_u)\cdot(\alpha_1-1)+\varepsilon_u\cdot(\gamma_1+\gamma_2)\cdot(\alpha_2-\alpha_1)\right]\cdot\frac{f_y}{u_{pw}}$$

$\theta_{p1w} = 0.019\,\text{rad}$

$$\theta_{uw} := \theta_{yw} + \frac{1}{4}\cdot\frac{d_b}{H_{col}}\cdot\left[(\varepsilon_y+\gamma_1\cdot\varepsilon_u)\cdot(\alpha_1-1)+\varepsilon_u\cdot(1+\gamma_1)\cdot(\alpha_3-\alpha_1)\right]\cdot\frac{f_y}{u_{pw}}$$

$\theta_{uw} = 0.025\,\text{rad}$

$$\Theta_w := \begin{bmatrix} 0 & \theta_{yw} & \theta_{pw} & \theta_{p1w} & \theta_{uw} \end{bmatrix}^T$$

Intermediate bond model:

$$\theta_{yi} := \frac{1}{4}\cdot\frac{d_b}{H_{col}}\cdot\varepsilon_y\cdot\frac{f_y}{u_{ei}}$$

$\theta_{yi} = 0.00039\,\text{rad}$

$$\theta_{pi} := \theta_{yi} + \frac{1}{4}\cdot\frac{d_b}{H_{col}}\cdot(\varepsilon_y+\gamma_1\cdot\varepsilon_u)\cdot(\alpha_1-1)\cdot\frac{f_y}{u_{pi}}$$

$\theta_{pi} = 0.00594\,\text{rad}$

$$\theta_{p1i} := \theta_{yi} + \frac{1}{4}\cdot\frac{d_b}{H_{col}}\cdot\left[(\varepsilon_y+\gamma_1\cdot\varepsilon_u)\cdot(\alpha_1-1)+\varepsilon_u\cdot(\gamma_1+\gamma_2)\cdot(\alpha_2-\alpha_1)\right]\cdot\frac{f_y}{u_{pi}}$$

$\theta_{p1i} = 0.0076\,\text{rad}$

$$\theta_{ui} := \theta_{yi} + \frac{1}{4}\cdot\frac{d_b}{H_{col}}\cdot\left[(\varepsilon_y+\gamma_1\cdot\varepsilon_u)\cdot(\alpha_1-1)+\varepsilon_u\cdot(1+\gamma_1)\cdot(\alpha_3-\alpha_1)\right]\cdot\frac{f_y}{u_{pi}}$$

$\theta_{ui} = 0.00993\,\text{rad}$

$$\Theta_i := \begin{bmatrix} 0 & \theta_{yi} & \theta_{pi} & \theta_{p1i} & \theta_{ui} \end{bmatrix}^T$$

Strong bond model:

$$\theta_{ys} := \frac{1}{4}\cdot\frac{d_b}{H_{col}}\cdot\varepsilon_y\cdot\frac{f_y}{u_{es}}$$

$\theta_{ys} = 0.00039\,\text{rad}$

$$\theta_{ps} := \theta_{ys} + \frac{1}{4}\cdot\frac{d_b}{H_{col}}\cdot(\varepsilon_y+\gamma_1\cdot\varepsilon_u)\cdot(\alpha_1-1)\cdot\frac{f_y}{u_{ps}}$$

$\theta_{ps} = 0.00317\,\text{rad}$

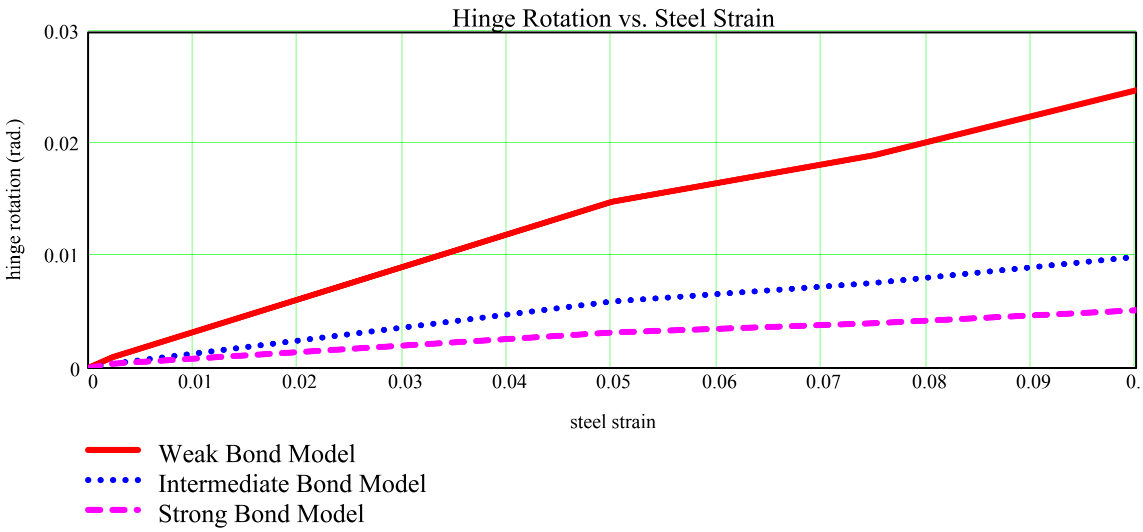
$$\theta_{p1s} := \theta_{ys} + \frac{1}{4}\cdot\frac{d_b}{H_{col}}\cdot\left[(\varepsilon_y+\gamma_1\cdot\varepsilon_u)\cdot(\alpha_1-1)+\varepsilon_u\cdot(\gamma_1+\gamma_2)\cdot(\alpha_2-\alpha_1)\right]\cdot\frac{f_y}{u_{ps}}$$

$\theta_{p1s} = 0.004\,\text{rad}$

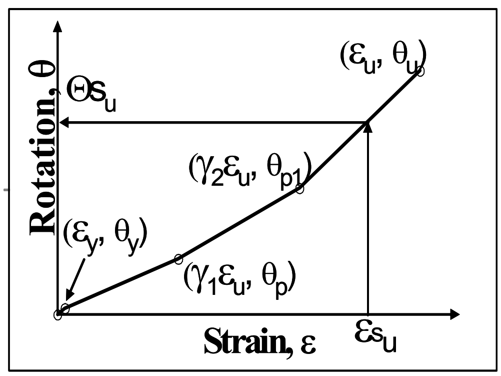
$$\theta_{us} := \theta_{ys} + \frac{1}{4}\cdot\frac{d_b}{H_{col}}\cdot\left[(\varepsilon_y+\gamma_1\cdot\varepsilon_u)\cdot(\alpha_1-1)+\varepsilon_u\cdot(1+\gamma_1)\cdot(\alpha_3-\alpha_1)\right]\cdot\frac{f_y}{u_{ps}}$$

$\theta_{us} = 0.00516\,\text{rad}$

$$\Theta_s := \begin{bmatrix} 0 & \theta_{ys} & \theta_{ps} & \theta_{p1s} & \theta_{us} \end{bmatrix}^T$$



Interpolate moment-curvature steel strains (εs_y, εs_n, εs_u) in steel-strain vs. rotation relationship to obtain (Θs_y, Θs_n, Θs_u)



Weak bond model:

Hinge rotation at My: $\theta_{yw} := \text{linterp}\left(\varepsilon_s, \Theta_w, \varepsilon s_y\right)$

Hinge rotation at Mn: $\theta_{nw} := \text{linterp}\left(\varepsilon_s, \Theta_w, \varepsilon s_n\right)$

Hinge rotation at Mu: $\theta_{uw} := \text{linterp}\left(\varepsilon_s, \Theta_w, \varepsilon s_u\right)$ $\quad \Theta h_w := \begin{bmatrix} 0 & \theta_{yw} & \theta_{nw} & \theta_{uw} \end{bmatrix}^T$

Intermediate bond model:

Hinge rotation at My: $\theta_{yi} := \text{linterp}\left(\varepsilon_s, \Theta_i, \varepsilon s_y\right)$

Hinge rotation at Mn: $\theta_{ni} := \text{linterp}\left(\varepsilon_s, \Theta_i, \varepsilon s_n\right)$

Hinge rotation at Mu: $\theta_{ui} := \text{linterp}\left(\varepsilon_s, \Theta_i, \varepsilon s_u\right)$ $\quad \Theta h_i := \begin{bmatrix} 0 & \theta_{yi} & \theta_{ni} & \theta_{ui} \end{bmatrix}^T$

Strong bond model:

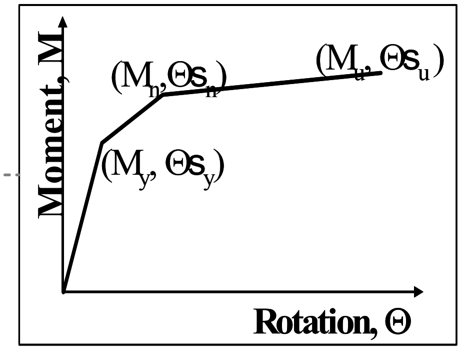
Hinge rotation at My: $\theta_{ys} := \text{linterp}\left(\varepsilon_s, \Theta_s, \varepsilon s_y\right)$

Hinge rotation at Mn: $\theta_{ns} := \text{linterp}\left(\varepsilon_s, \Theta_s, \varepsilon s_n\right)$

Hinge rotation at Mu: $\theta_{us} := \text{linterp}\left(\varepsilon_s, \Theta_s, \varepsilon s_u\right)$ $\quad \Theta h_s := \begin{bmatrix} 0 & \theta_{ys} & \theta_{ns} & \theta_{us} \end{bmatrix}^T$

Critical Moments: $\quad Mh := Mynu_{col}$

Plot Moment-Rotation relationship for hinge spring element ($\Theta s_y, M_y$) ($\Theta s_n, M_n$) ($\Theta s_u, M_u$)

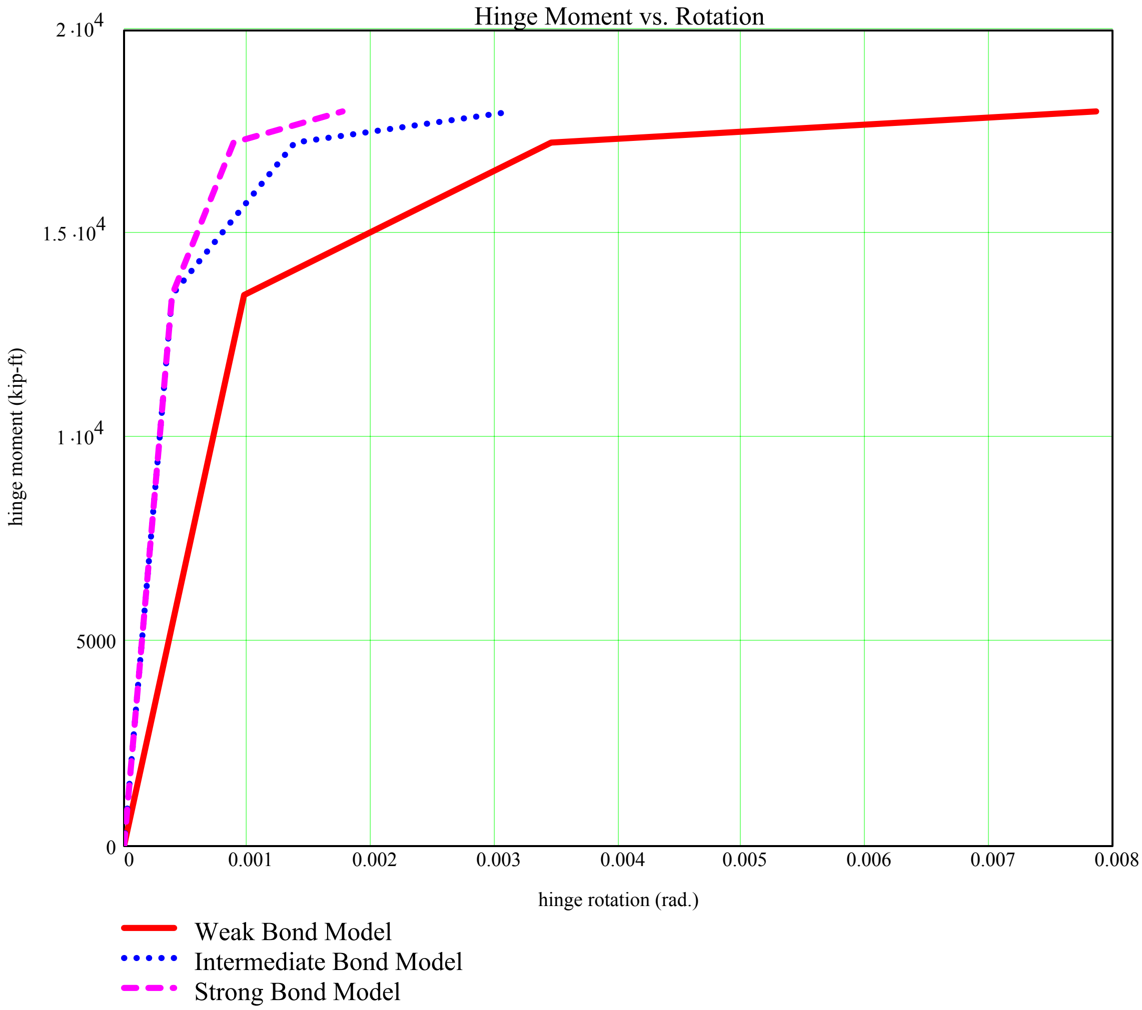


Weak bond model: $\Theta h_w = \begin{bmatrix} 0 \\ 0.00097 \\ 0.00346 \\ 0.00787 \end{bmatrix} \cdot rad$

Intermediate bond model: $\Theta h_i = \begin{bmatrix} 0 \\ 0.00039 \\ 0.00138 \\ 0.00315 \end{bmatrix} \cdot rad$

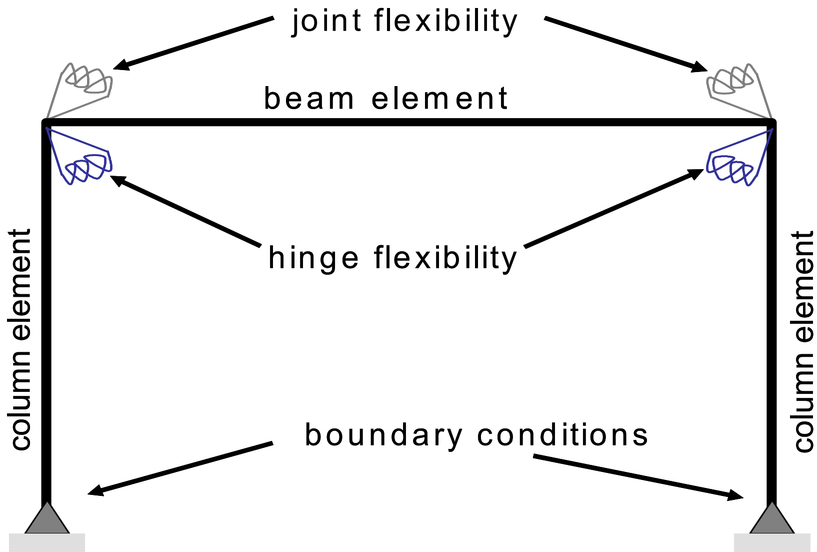
Strong bond model: $\Theta h_s = \begin{bmatrix} 0 \\ 0.00039 \\ 0.00089 \\ 0.00177 \end{bmatrix} \cdot rad$

Critical Moments: $Mh = \begin{bmatrix} 0 \\ 13511 \\ 17248 \\ 18010 \end{bmatrix} \cdot kip \cdot ft$



Proceed to calculating structural displacement capacities and demands using the recommended joint and hinge models.

Select joint and hinge cathegory and model.
Incorporate rotational springs at the joint nodes and column ends.



Static-Capacity calculations:
Perform nonlinear static pushover analysis to determine drift capacity.
Dynamic-Demand calculations:
1. perform nonlinear dynamic analyses with design-level ground motions to determine drift demands.
or 2. Calculate effective elastic stiffness of bridge bent which accounts for hinge and joint flexibilities. Use elastic design spectra to determine drift demands.

SUMMARY

Geometry:

Column length:	$L_{col} = 36\ ft$	Beam length:	$L_{beam} = 36\ ft$
Column diameter:	$H_{col} = 6.5\ ft$	Beam depth:	$H_{beam} = 8\ ft$
Column long. steel ratio:	$\rho_{col} = 1.75 \cdot \%$	Beam width:	$B_{beam} = 6.5\ ft$
Column long. steel diameter:	$d_b = 1.693 \cdot in$		
Superstructure Weight:	$Weight = 3000 \cdot kip$		

Joint Analysis III

determine joint-boundary forces from Pushover analysis (compression column).
This is the most accurate analysis

Joint shear stress demand: $v_{jIII} = 0.138 \cdot f_c$ $\quad v_{jIII} = 10.2 \cdot \sqrt{f_c \cdot psi}$

Factored nominal joint shear strength:

Weak joint:	$\phi \cdot vn_{weak} = 4.25 \cdot \sqrt{f_c \cdot psi}$	Intermediate joint:	$\phi \cdot vn_{int} = 6.375 \cdot \sqrt{f_c \cdot psi}$
Moderate joint:	$\phi \cdot vn_{mod} = 4.25 \cdot \sqrt{f_c \cdot psi}$	Strong joint:	$\phi \cdot vn_{strongIII} = 10.657 \cdot \sqrt{f_c \cdot psi}$

$v_j < 0.5\ (\phi v_n)$ — 1: Beam-column joint can be assumed rigid

$0.5\ (\phi v_n)$ # v_j # ϕv_n — 2: Beam-column joint can be modeled as elastic member. Yielding of beam-column joints will occur without measurable strength loss.

v_j $ ϕv_n — 3: Strength and stiffness degradation can be expected

$v_{jIII} = 4.823 \cdot (0.5 \cdot \phi \cdot vn_{weak})$ $\quad v_{jIII} = 2.411 \cdot \phi \cdot vn_{weak}$

$v_{jIII} = 4.823 \cdot (0.5 \cdot \phi \cdot vn_{mod})$ $\quad v_{jIII} = 2.411 \cdot \phi \cdot vn_{mod}$

$v_{jIII} = 3.215 \cdot (0.5 \cdot \phi \cdot vn_{int})$ $\quad v_{jIII} = 1.608 \cdot \phi \cdot vn_{int}$

$v_{jIII} = 1.923 \cdot (0.5 \cdot \phi \cdot vn_{strongIII})$ $\quad v_{jIII} = 0.962 \cdot \phi \cdot vn_{strongIII}$

Weak & moderate joint:	$v_j > \phi \cdot v_n$	strength and stiffness degradation can be expected
Intermediate joint:	$v_j > \phi \cdot v_n$	strength and stiffness degradation can be expected
Strong joint:	$0.5 \cdot \phi \cdot v_n < v_j < \phi \cdot v_n$	Yielding of beam-column joint will occur without strength loss.

Weak Joint Model:

$\Theta j_w^T = (0 \quad 0.00015 \quad 0.00037 \quad 0.01)\cdot rad$

$Mj_w^T = (0 \quad 12634 \quad 18048 \quad 0)\cdot kip\cdot ft$

Moderate Joint Model:

$\Theta j_m^T = (0 \quad 0.00015 \quad 0.00037 \quad 0.01)\cdot rad$

$Mj_m^T = (0 \quad 12634 \quad 18048 \quad 18048)\cdot kip\cdot ft$

Intermediate Joint Model:

$\Theta j_i^T = (0 \quad 0.00021 \quad 0.00126 \quad 0.1)\cdot rad$

$Mj_i^T = (0 \quad 18048 \quad 27072 \quad 27099)\cdot kip\cdot ft$

Strong Joint Model:

$\Theta j_s^T = \left[0 \quad 3.158\cdot 10^{-4} \quad 1.645\cdot 10^{-3} \quad 0.1\right]\cdot rad$

$Mj_s^T = (0 \quad 27072 \quad 38467 \quad 48083)\cdot kip\cdot ft$

Elastic Joint Model:

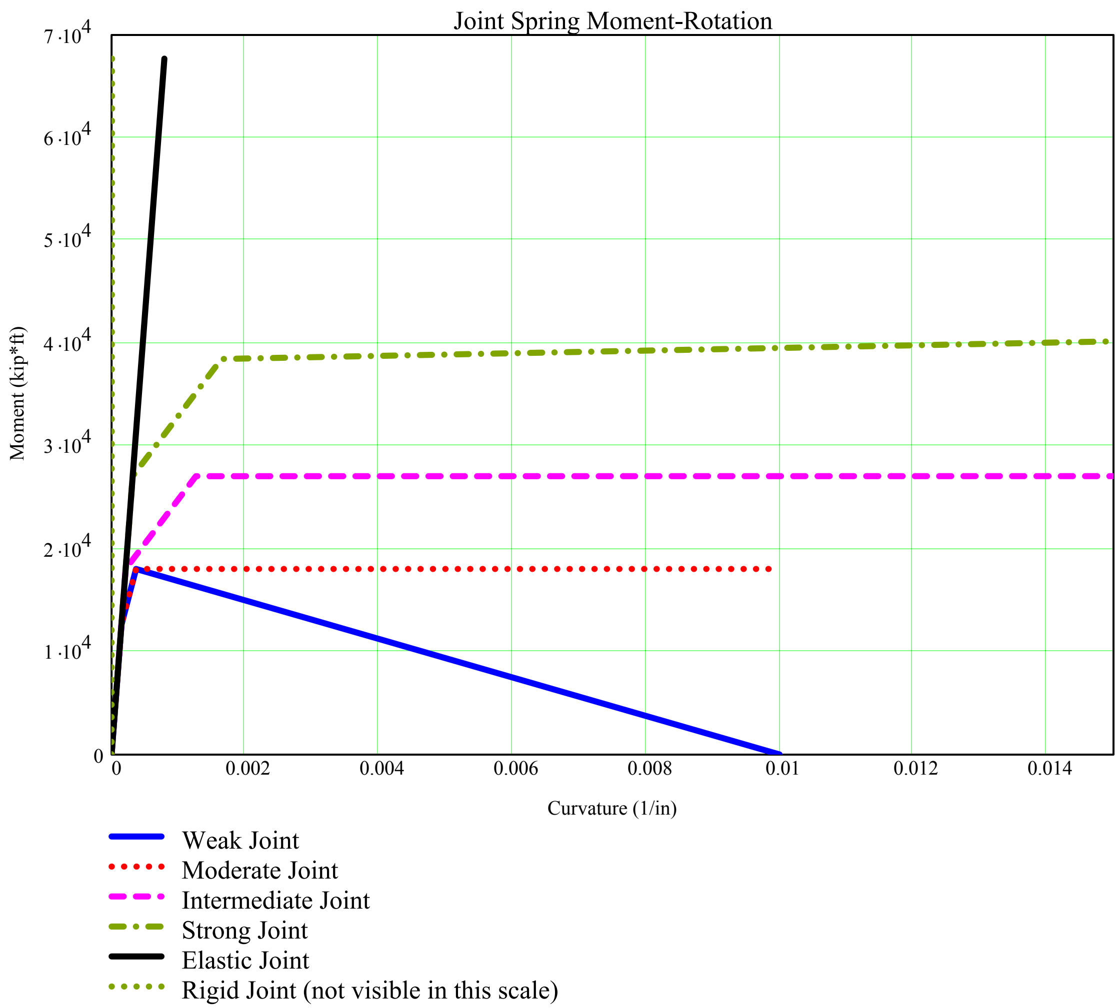
$\Theta j_e^T = (0 \quad 0.00032 \quad 0.00063 \quad 0.00079)\cdot rad$

$Mj_e^T = (0 \quad 27072 \quad 54144 \quad 67680)\cdot kip\cdot ft$

Rigid Joint Model:

$\Theta j_r^T = \left[0 \quad 3.158\cdot 10^{-6} \quad 6.316\cdot 10^{-6} \quad 7.895\cdot 10^{-6}\right]\cdot rad$

$Mj_r^T = (0 \quad 27072 \quad 54144 \quad 67680)\cdot kip\cdot ft$



HINGE MODEL

Moment-Curvature Data (yield, nominal & ultimate points):

$$\Phi ynu_{col}^{T} = \left[0 \quad 6.012\cdot 10^{-5} \quad 1.975\cdot 10^{-4} \quad 8.589\cdot 10^{-4} \right]\cdot\frac{1}{in}$$

$$Mynu_{col}^{T} = (0 \quad 13511 \quad 17248 \quad 18010)\cdot kip\cdot ft$$

Select Bond-Stress Model

"Strong" bond model :	"Intermediate" bond model:	"Weak" bond model:	bar stress
$u_e=30\sqrt{f'_c}$	$u_e=30\sqrt{f'_c}$	$u_e=12\sqrt{f'_c}$	pre-yield
$u_p=30\sqrt{f'_c}$	$u_p=15\sqrt{f'_c}$	$u_p=6\sqrt{f'_c}$	post-yield

Moment-Rotation Charactristics of Hinge Model:

Weak bond model: **Intermediate bond model:** **Strong bond model:** **Critical Moments:**

$$\Theta h_{w} = \begin{bmatrix} 0 \\ 0.00097 \\ 0.00346 \\ 0.00787 \end{bmatrix} rad \qquad \Theta h_{i} = \begin{bmatrix} 0 \\ 0.00039 \\ 0.00138 \\ 0.00315 \end{bmatrix} rad \qquad \Theta h_{s} = \begin{bmatrix} 0 \\ 0.00039 \\ 0.00089 \\ 0.00177 \end{bmatrix} rad \qquad Mh = \begin{bmatrix} 0 \\ 13511 \\ 17248 \\ 18010 \end{bmatrix} kip\cdot ft$$